Fluid flow through cattle hide: an experimental permeability study

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The permeability of chrome-tanned cattle hide was determined separately with distilled water and an 89-weight percent aqueous glycerol solution in the light of a simplified Darcy's law. The resultant permeability has been found to depend on the location from which the sample is taken on the original hide. The frequency distribution of the values of the permeability has been shown to be lognormal, thereby giving rise to a two parameter-description of the distribution.

1. Introduction

Hide and leather are naturally occurring porous materials. At a macroscopic level, their pore structure is essentially isotropic; however, when considered at the microscopic level, it is anisotropic. Permeability is perhaps the most significant parameter for describing flow of fluids through such materials. Hide and leather are known to exhibit a statistical distribution of properties for samples obtained from various locations on the hide. The distribution of the measured values of permeability is characterized in the current work for chrome-tanned cattle hide.

A previous work [1] represents the permeability of leather as a range of values; however, neither the most probable value nor the frequency distribution of the permeability is discernable from such a range. Hence, specifying a range for the permeability has only limited utility for designing processes involving flow of fluids through leather [2].

2. Theoretical

Consider fluid flow through a porous column having the cross-sectional area, A, and the length, L, which is subjected to the pressure difference, $\Delta P = P_2 - P_1$, or the pressure drop, $(-\Delta P) = P_1 - P_2$ (Fig. 1). The macroscopic momentum and energy balances portray the flow of fluid through the thickness of the leather.

2.1. Macroscopic momentum balance

The macroscopic equations for conservation of momentum express the pressure gradient, or head, in terms of the average drag, $\boldsymbol{D}$, as a function of the microscopic velocity field, $\boldsymbol{u}$ [3]. For uniform average flow of a homogeneous and incompressible fluid, the steady-state macroscopic momentum balance yields [4]

$$\frac{\mathrm{d}}{\mathrm{d}t}\vec{P}_{tot} = 0 = \rho_1\langle \nu_1^2\rangle\vec{S}_1 - \rho_2\langle \nu_2^2\rangle\vec{S}_2 + P_1\vec{S}_1 - P_2\vec{S}_2 - \vec{F} + m\vec{g} \quad (1)$$

where $\vec{S}_i$ is the vector whose magnitude is equal to the cross-sectional area at surface i in the direction of the fluid flow; $\vec{F}$, the force exerted by the fluid on the sample; and m, the total mass of the fluid contained between planes 1 and 2 separated by a distance of L.[4]

When the cross-sectional area, A, and the porosity, ϵ, are constant, the z-component of $\vec{S}_i$, which is the cross-sectional area available to flow, is

$$\vec{S} = \varepsilon A = \vec{S}_1 = \vec{S}_2$$

Since the flow is considered to be incompressible, we have

$$\rho_1 = \rho_2 = \rho$$

Thus, the macroscopic mass balance for steady-state flow becomes

$$\rho S(\langle\nu_1\rangle - \langle\nu_2\rangle) = 0$$

or

$$\nu_1 = \nu_2 = \nu \quad (2)$$

Consequently, Equation 1 reduces to

$$\vec{F} = (P_1 - P_2)S + m\vec{g} = \varepsilon A[-\Delta\vec{P} - \rho L\vec{g}] \quad (3)$$

By definition, the force exerted by the fluid on the unit volume of the sample can be regarded as the average drag, $\boldsymbol{D}$; therefore, Equation 3 further reduces to

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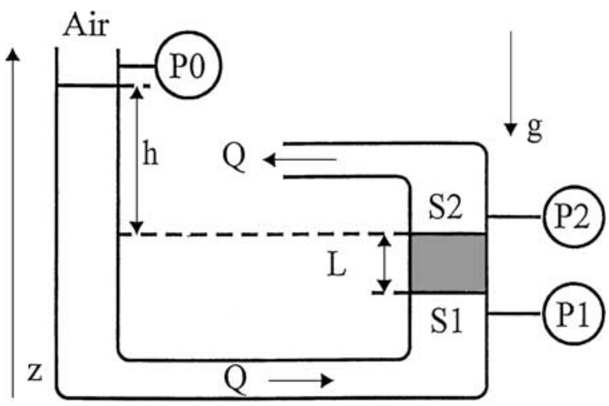

Figure 1 Schematic of the experimental setup indicating the flow path and the quantities required for calculating the permeability indicated.

$$\boldsymbol{D} = \frac{\vec{F}}{AL} = \frac{-\varepsilon \Delta \vec{P}}{L} - \varepsilon \rho \vec{g}$$

or

$$\frac{\boldsymbol{D}}{\rho \vec{g} \varepsilon} = \frac{-1}{\rho \vec{g} L}[\Delta \vec{P} + \rho \vec{g} L] \quad (4)$$

Darcy's law stipulates that the average drag, $\boldsymbol{D}$, varies linearly with the specific discharge, q, defined as the volumetric flow rate per unit overall surface area including both the solid and fluid phases. As a result,

$$\boldsymbol{q} = \frac{\boldsymbol{Q}}{A} = \boldsymbol{KD} \quad (5)$$

Note that $\boldsymbol{q}$ is the superficial velocity; it is a scalar quantity for one-dimensional steady flow. Combining Equations 4 and 5 gives rise to

$$\frac{\boldsymbol{Q}}{A} = -\frac{\boldsymbol{K}\varepsilon}{L}[\Delta P + \rho g L] \quad (6)$$

The proportional constant, $\boldsymbol{K}$, in the above expression depends only on the structure of the medium and the properties of the fluid but does not depend on the pressure drop across the sample, $(-\Delta P)$; it is termed the hydraulic conductivity when the fluid is water at 25°C. Moreover, $\boldsymbol{K}$ can be replaced by the scalar, K, for an isotropic medium. Substituting K with k/μ in Equation 6 gives rise to Darcy's law [3, 5, 6]; i.e.,

$$\frac{Q}{A} = -\left(\frac{k}{\mu}\right)\left(\frac{\varepsilon}{L}\right)[\Delta P + \rho g L] \quad (7)$$

where k is defined as the specific permeability or intrinsic conductivity; and μ, the viscosity. Upon rearrangement,

$$-\Delta P = P_1 - P_2 = \rho g L + \frac{Q\mu L}{k\varepsilon A} \quad (8)$$

The unit of k is cm^2, but its typical values are small. Hence, a unit of permeability, termed the "darcy", has been defined as 0.987×10^{-8} cm^2. A pressure difference of 1 atmosphere will produce a flow rate of 1 cubic centimeter per second through a cube having sides 1 cm in length and a permeability of 1 darcy when the fluid has a viscosity of 1 centipoise.

The range of validity of Darcy's law depends on the Reynolds number of flow; it must be such that the flow is totally laminar [7]. The available experimental data indicate that a Reynolds number of 1.0 appears to be the upper limit for the applicability of Darcy's law. Nevertheless, reasonable agreement with experimental data has been reported for the Reynolds numbers as large as 10^3. Darcy's law also demands that steady-state flow prevails, which can be sustained by applying a constant pressure to the fluid or by maintaining a constant hydrostatic head in the tank. Naturally, local and/or dynamic inertia forces are negligibly small for any totally laminar flow under the conditions of macroscopic steady-state flow.

2.2. Macroscopic energy balance

The general expression of Darcy's law, Eqution 8, must be transformed to calculate permeabilities from the experimental data obtained with the permeameter described in the Experimental section. A general schematic of the flow path (Fig. 1) illustrates the method by which the constant pressure, P_1, is applied. The pressures measured are those of the air above the fluid in the tank, P_0, and of the fluid after flowing through the sample, P_2. While Equation 8 is based on the pressures, P_1 and P_2, the design of the permeameter prevents the measurement of the pressure, P_1. The pressure difference, ΔP, i.e., $(P_2 - P_1)$, however, can be estimated from the macroscopic mechanical energy balance and the pressure difference, ΔP_{sys}, i.e., $(P_2 - P_0)$, which is experimentally measurable. For steady-state flow of an incompressible fluid with a flat velocity profile, the macroscopic mechanical energy balance simplifies to [4]

$$\Delta\left[\left(\frac{1}{2}\langle v\rangle^2 + \hat{\Phi} + \frac{P}{\rho}\right)w\right] + \hat{W} + E_v = 0 \quad (9)$$

Since $v_1 = v_2 = v$ from Equation 2 and no work is performed by the system on its surroundings, the terms, $\Delta\lfloor(1/2)\langle\bar{v}\rangle^2\rfloor$ and $\hat{W}$, in Equation 9 vanish; as a result,

$$\Delta\hat{\Phi} + \frac{\Delta P}{\rho} + \hat{E}_v = 0 \quad (10)$$

For the segment between planes 0 and 2,

$$\Delta\hat{\Phi} = \hat{\Phi}_2 - \hat{\Phi}_0 = gh$$

and thus, we have from Equation 10

$$\rho g h + (P_2 - P_0) + \rho \hat{E}_{v_{02}} = 0$$

or

$$\rho \hat{E}_{v_{02}} = -\rho g h - (P_2 - P_0) \quad (11)$$

For the segment between planes 1 and 2,

$$\Delta\hat{\Phi} = \hat{\Phi}_2 - \hat{\Phi}_1 = gL$$

and thus, we have from Equation 10

$$\rho gL + (P_2 - P_1) + \rho\hat{E}_{v_{12}} = 0$$

or

$$\rho\hat{E}_{v_{12}} = -\rho gL + (P_1 - P_2) \tag{12}$$

Substituting $(P_1 - P_2)$ from Equation 8 into the above equation gives

$$\rho\hat{E}_{v_{12}} = \frac{Q\mu L}{k\varepsilon A} \tag{13}$$

Since

$$\rho[\hat{E}_{v_{01}} + \hat{E}_{v_{12}}] = \rho\hat{E}_{v_{02}},$$

we obtain

$$\rho\hat{E}_{v_{12}} = \rho\hat{E}_{v_{02}} - \rho\hat{E}_{v_{01}}$$

Substituting $\rho\hat{E}_{v_{02}}$ from Equation 11 and $\rho\hat{E}_{v_{12}}$ from Equation 13 into this expression yields

$$\frac{Q\mu L}{k\varepsilon A} = [-\rho gh - (P_2 - P_0)] - \rho\hat{E}_{v_{01}}$$

which, in turn, leads to

$$k = \frac{Q\mu L}{A\varepsilon\{[-\rho gh - (P_2 - P_0)] - \rho\hat{E}_{v_{01}}\}} \tag{14}$$

Note that $\Delta P_{sys} = P_2 - P_0$. Under the conditions of creeping flow for which Darcy's Law is valid, the frictional loss or dissipation, $\hat{E}_{v_{01}}$, is negligibly small. Hence, Equation 14 reduces to

$$k = \frac{Q\mu L}{A\varepsilon(-\rho gh - \Delta P_{sys})} \tag{15}$$

Since ΔP_{sys} is measurable, Equation 15 can be written more compactly as

$$\kappa' = -\frac{Q\mu L}{A(\Delta P_{tot})} \tag{16}$$

where ΔP_{tot} is the total pressure drop, i.e.,

$$\begin{aligned}\Delta P_{tot} &= (P_{tot})_2 - (P_{tot})_0 \\ &= (P_2 + \rho gh) - P_0 \end{aligned} \tag{17}$$

Obviously, all the quantities in the right-hand side of Equation 16 are measurable or can be predetermined. The superficial permeability, κ', in the left-hand side lumps k and ε as a product because of the appreciable uncertainty involved in predetermining ε through either measurement or estimation [8].

3. Experimental

The hides tested in this study were cut from number 2 chrome-tanned cattle hides [9]. The values of permeability were measured with two fluids; the first was distilled water with a viscosity of 0.8904 cp [10], and the second was an aqueous solution containing 89 weight percent of glycerol with a viscosity of 100 cp [11].

The permeability tests were performed with a Soiltest model K-670 miniature high-pressure permeameter apparatus. Although capable of maintaining a pressure of 100 psi, the standard unit was supplied with gauges having a maximum reading of 60 psi. The permeameter was designed to test cylindrical soil and concrete specimens approximately 1.3 inches in diameter and 2.8 inches in length; therefore, adding a gasket to seal the thinner hide samples, thus preventing the fluid from bypassing the sample, modified the specimen chamber provided with the permeameter. This gasket was not replaced for the duration of the tests to ensure uniform results.

The tank containing the fluid was approximately 4 inches in diameter and 20 inches in length. Applying pressure to the fluid with compressed air induced flow of fluid from this tank. A constant relieving-type regulator controlled the pressure. In practice, the pressure applied from the compressed air tank is typically much larger in magnitude than the pressure exerted from the fluid in the tank or the pressure drop due to frictional loss. The pressure drop due to frictional losses was estimated to be a maximum of five orders of magnitude smaller than the measured pressure drop, as already stipulated in the preceding section in connection with Darcy's law. The frictional losses were, therefore, neglected in any calculations. Only the pressure applied from the compressed air tank and the hydrostatic head were included in the calculations, as indicated by Equation 18.

The permeability of chrome-tanned cattle hide and its variation were evaluated under four different scenarios.

1. Thirteen replicate samples cut from each of four equal-sized areas on a single hide were mounted such that their grain sides faced toward the direction of flow of distilled water at 25°C.
2. Thirteen samples prepared in the same manner as described above were mounted such that their grain sides faced away from the direction of flow of distilled water at 25°C.
3. Eight replicate samples cut from a single hide sample were mounted such that their grain sides faced away from the direction of flow of distilled water at 25°C. Various levels of the pressure exceeding that of the liquid in the reservoir tank were applied to assess the effect of applied pressure.
4. Four replicate samples cut from four different hide samples were mounted such that their grain sides faced toward the direction of flow of an 89-weight percent aqueous glycerol solution to study the impact of the fluid's viscosity, density, and surface tension.

4. Results and discussion

On a macroscopic scale, chrome-tanned cattle hide is an isotropic, homogeneous, porous medium. Moreover,

TABLE I Statistics for each group of test conditions

Group	Mean permeability (darcies)	Standard deviation (darcies)	Confidence interval (95%) (darcies)
1. Grain up	0.0257	0.0163	0.0356
2. Grain down	0.0182	0.0095	0.0208
3. Pressure dependence	0.0346	0.0027	0.0064
4. Glycerol solution	0.00474	0.00191	0.00607

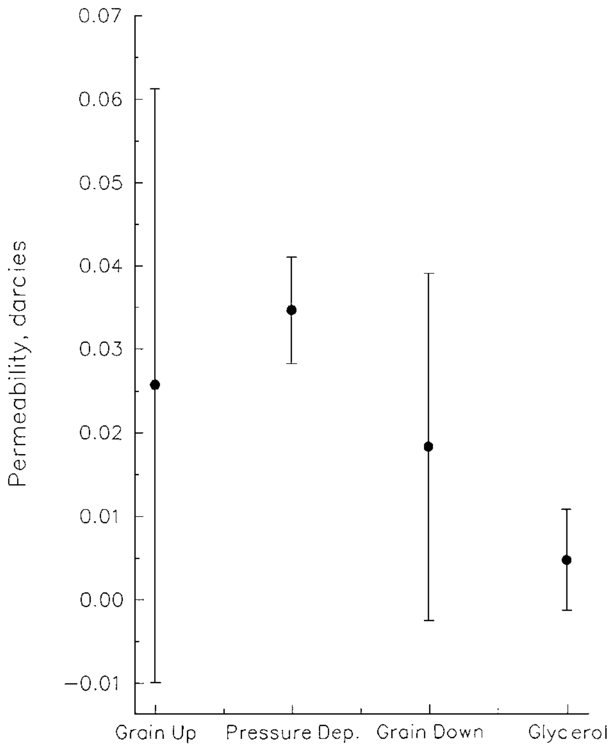

Figure 2 Arithmetic mean permeabilities and their associated 95% confidence intervals for all hide samples.

the flow rates of the measurement fluid were sufficiently low to ensure creeping flow in the current work. Thus, Darcy's law should be applicable for recovering the permeability of cattle hide. The resultant permeabilities from each set of test conditions are summarized in Table I along with the 95 percent confidence intervals associated with each group of data.

Comparison of the mean values and confidence intervals of the permeabilities evaluated with Darcy's law for all groups plotted in Fig. 2 reveals that the measured values within each group indeed remain invariant within the 95% confidence intervals, thereby establishing that various flow conditions give rise to essentially identical permeabilities.

The permeabilities measured as a function of the applied pressure are displayed in Fig. 3. The error bars reflect the maximum experimental error of each measurement. Since the error bars for each data point overlap, the mean permeability of the entire data set is representative. These data illustrate that the permeability is essentially independent of the applied pressure and confirm the applicability of Darcy's law for fluid flow through cattle hide.

A histogram of permeability encompassing all data sets is portrayed in Fig. 4. Only the mean permeability

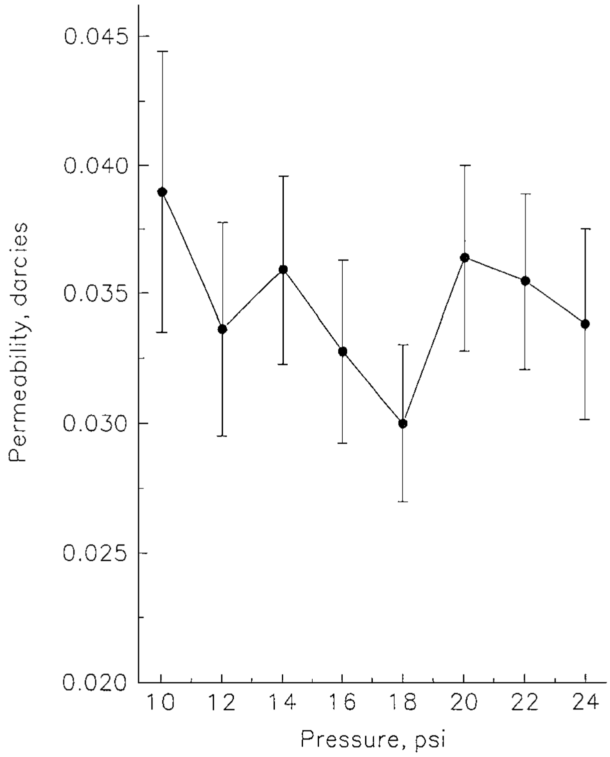

Figure 3 Permeability and the maximum experimental error associated with each measurement of a single cattle hide sample as a function of applied pressure.

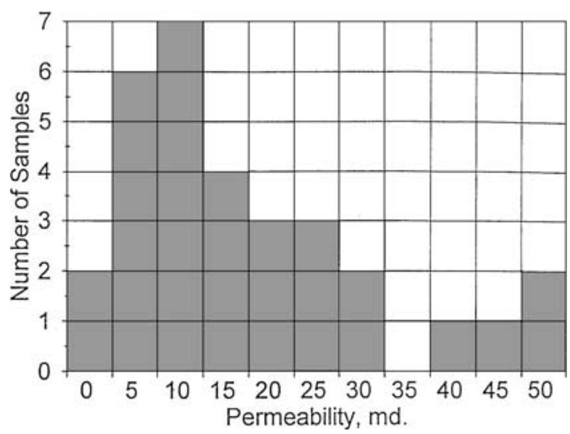

Figure 4 Histogram of the permeabilities for 31 cowhide samples at intervals of 5 md.

from the eight pressure dependence tests is included in the histogram because all the data have been obtained with a single hide sample. In constructing the histogram, a constant interval reveals the details of the distribution without excessive spikes and valleys. Although a gap in permeability exists between 35 and 40 md, choosing a larger interval would result in excessive smoothing of the distribution.

The histogram mentioned above is decidedly skewed towards lower permeabilities, i.e., to the right; the mean permeability is clearly not the most probable value. To address the skewness of this permeability and to better describe the distribution quantitatively, distribution intervals have been chosen so that they correspond to a geometric progression of permeability [12, 13]; with a progression factor of $\sqrt{3}$, the resulting histogram approximates a normal distribution, as discernible in

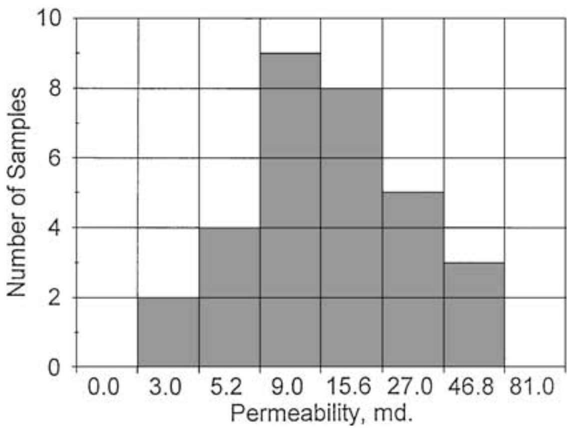

Figure 5 Histogram of the permeabilities for 31 cowhide samples at intervals following the geometrical progression with a factor of $\sqrt{3}$.

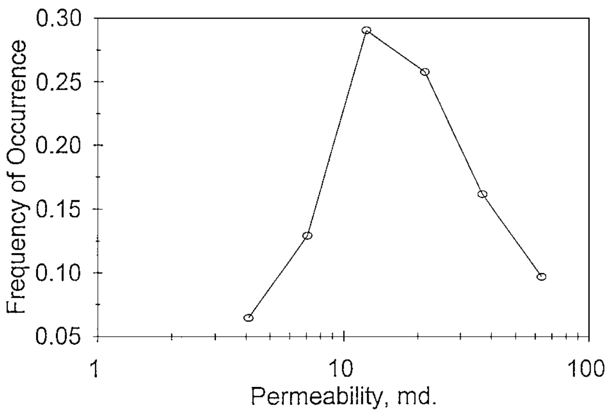

Figure 6 Experimentally measured permeabilities plotted as a log-normal frequency of occurrence distribution with a logarithmic scale for the mean permeability of each interval on the x-axis.

Fig. 5. Nevertheless, skewness to lower permeabilities still persists. This skewness has been eliminated by plotting the frequency of occurrence as a function of the logarithm of the mean permeability [13–15], thereby yielding the lognormal distribution presented in Fig. 6, which can be expressed as

$$y = \frac{1}{\sigma_z\sqrt{2\pi}} \exp\left[-\frac{(z-\bar{z})^2}{2\sigma_z^2}\right] \tag{19}$$

where $z = \log(x)$, in which x is the measured permeability of the cowhide. The arithmetic mean of z, i.e., $\bar{z}$, yields the most probable value of the permeability, specifically 15.8 md; the corresponding standard deviation is 2.1 md.

5. Conclusions

The permeability of chrome-tanned cattle hide to water and to an aqueous glycerol solution have been determined on the basis of a simplified Darcy's law derived from macroscopic momentum and mechanical energy balances. Furthermore, the permeabilities measured have been found to be essentially independent of the applied pressure or the fluid viscosity under the conditions of the current study.

Within a confidence interval of 95 percent, the permeabilities measured under all test conditions are statistically equivalent. The values of the permeability of chrometanned cattle hide are log-normally distributed; the most probable value and the standard deviation are 15.8 md and 2.1 md, respectively.

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